Subspace Approach to Identification of Linear Repetitive Processes

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Outline

The aim of this paper – to propose a new approach to the identification of the LRPs based on subspace algorithms.

The order of a LRP and the unknown process matrices are determined based on the input and output sequences of the actual pass and the output sequence of the previous pass

Subspace methods are applied to identify MIMO state-space systems using numerically robust computational tools such as the QR factorization and the singular-value decomposition (SVD) and numerically robust geometrical operations such as orthogonal and oblique projections:

- No canonical parametrization is needed
- No nonlinear optimization is performed
- Reliable state-space models for complex MIMO systems are derived directly from the input-output data
- Computational complexity is modest in comparison with the well-known prediction error methods

State-space model of a discrete linear repetitive process

$$x_{k+1}(p+1) = Ax_{k+1}(p) + B_0 y_k(p) + Bu_{k+1}(p) \tag{1}$$

$$y_{k+1}(p) = Cx_{k+1}(p) + D_0 y_k(p) + Du_{k+1}(p) \tag{2}$$

$0 \leq p \leq \alpha - 1 \in \mathsf{Z}_+$ is – independent spatial or temporal variable,
$k \in \mathsf{Z}_+$ – the current pass number,
$x_k(p) \in \mathsf{R}^n$ – the state vector,
$y_k(p) \in \mathsf{R}^l$ – the pass profile (output) vector,
$u_k(p) \in \mathsf{R}^m$ – the input vector,
A, B, B_0, C, D, D_0 – matrices of appropriate dimensions.
Bboundary conditions:

$$x_{k+1}(0) = d_{k+1} \tag{3}$$

$$y_0(p) = f(p) \tag{4}$$

$d_{k+1} \in \mathsf{R}^n$ – a vector with known constant entries, $f(p) \in \mathsf{R}^l$ are known functions of p.

Identification Problem

Given α measurements of the input $u_{k+1}(p)$ and the outputs $y_k(p)$ and $y_{k+1}(p)$ generated by the LRP (1)-(2) determine its order and the LRP matrices A, B, B_0, C, D and D_0 up to within a similarity transformation

Following Theorem 1 (P. Van Overschee, B. De Moor) the state-space model (1)-(2) can be reformulated in a matrix form

$$Y_p = \Gamma_i X_p + H_i U_p \tag{5}$$

$$Y_f = \Gamma_i X_f + H_i U_f \tag{6}$$

$$X_f = A^i X_p + \Delta_i U_p \tag{7}$$

Input block Hankel matrix

$$
U_{0|2i-1} = \left[\begin{array}{lll}
u_{k+1}(0) & \dots & u_{k+1}(j-1) \\
y_k(0) & \dots & y_k(j-1) \\
\dots & \dots & \dots \\
u_{k+1}(i-1) & \dots & u_{k+1}(i+j-2) \\
y_k(i-1) & \dots & y_k(i+j-2) \\
\hline
u_{k+1}(i) & \dots & u_{k+1}(i+j-1) \\
y_k(i) & \dots & y_k(i+j-1) \\
u_{k+1}(i+1) & \dots & u_{k+1}(i+j) \\
y_k(i+1) & \dots & y_k(i+j) \\
\dots & \dots & \dots \\
u_{k+1}(2i-1) & \dots & u_{k+1}(2i+j-1) \\
y_k(2i-1) & \dots & y_k(2i+j-2)
\end{array}\right]
$$

$$
= \left[\begin{array}{c} U_{0|i-1} \\ \hline U_{i|2i-1} \end{array}\right] = \left[\begin{array}{c} U_p \\ \hline U_f \end{array}\right]
$$

$$
= \left[\begin{array}{c} U_{0|i} \\ \hline U_{i+1|2i-1} \end{array}\right] = \left[\begin{array}{c} U_p^+ \\ \hline U_f^- \end{array}\right]
$$

Output block Hankel matrix

$$Y_{0|2i-1} = \left[\begin{array}{ccc} y_{k+1}(0) & \dots & y_{k+1}(j-1) \\ \dots & \dots & \dots \\ y_{k+1}(i-1) & \dots & y_{k+1}(i+j-2) \\ \hline y_{k+1}(i) & \dots & y_{k+1}(i+j-1) \\ y_{k+1}(i+1) & \dots & y_{k+1}(i+j) \\ \dots & \dots & \dots \\ y_{k+1}(2i-1) & \dots & y_{k+1}(2i+j-1) \end{array}\right]$$

$$= \left[\begin{array}{c} Y_{0|i-1} \\ \hline Y_{i|2i-1} \end{array}\right] = \left[\begin{array}{c} Y_p \\ \hline Y_f \end{array}\right]$$

$$= \left[\begin{array}{c} Y_{0|i} \\ \hline Y_{i+1|2i-1} \end{array}\right] = \left[\begin{array}{c} Y_p^+ \\ \hline Y_f^- \end{array}\right]$$

The number of block rows i should be larger than the maximum order of the LRP

Block Hankel matrices W_p and W_p^+

$$W_p = \begin{bmatrix} U_p \\ Y_p \end{bmatrix}$$

$$W_p^+ = \begin{bmatrix} U_p^+ \\ Y_p^+ \end{bmatrix}$$

The state-sequence matrix X_i

$$X_i = \begin{bmatrix} x_{k+1}(i) \dots x_{k+1}(i+j-1) \end{bmatrix}$$

The extended observability matrix Γ_i and the reversed extended controllability matrix Δ_i:

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ \dots \\ CA^{i-1} \end{bmatrix}$$

$$\Delta_i = \begin{bmatrix} A^{i-1}[B\ B_0] \dots A[B\ B_0][B\ B_0] \end{bmatrix}$$

Assumptions: $\{A, C\}$ – observable, $\{A, [B\ B_0]\}$ – controllable

The lower block triangular Toeplitz matrix H_i

$$H_i = \begin{bmatrix} [D\ D_0] & 0 & \dots & 0 \\ C[B\ B_0] & [D\ D_0] & \dots & 0 \\ CA[B\ B_0] & C[B\ B_0] & \dots & 0 \\ \dots & \dots & \dots & \dots \\ CA^{i-2}[B\ B_0] & CA^{i-3}[B\ B_0] & \dots & [D\ D_0] \end{bmatrix}$$

Deterministic Process Identification

Assumptions:

A1. The augmented input $[u_{k+1}(p)\ y_k(p)]$ is persistently exciting of order $2i$

A2. The intersection of the row space of U_f and the row space of X_p is empty

The unknown LRP matrices A, B, B_0, C, D and D_0 can be computed based on the results of Theorem 2 (P. Van Overschee, B. De Moor)

The order of the process (1)-(2) can be found based on inspection of the singular value decomposition of the matrix $W_1\mathcal{O}_iW_2$

Algorithm 1

1. Calculate the oblique projections

$$\mathcal{O}_i = Y_f /_{U_f} W_p \tag{8}$$

2. Calculate the singular value decomposition

$$W_1 \mathcal{O}_i W_2 = [U_1 \; U_2] \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \tag{9}$$

$W_1 \in \mathrm{R}^{li \times li}$ and $W_2 \in \mathrm{R}^{j \times j}$ – the user defined weighting matrices

3. Find the order of the LRP (1)-(2) based on inspection of the singular values in S_1

4. Determine the extended observability matrix Γ_i

$$\Gamma_i = W_1^{-1} U_1 S_1^{1/2} T \tag{10}$$

$T \in \mathrm{R}^{n \times n}$ – an arbitrary non-singular similarity transformation matrix

Algorithm 1

5. Calculate the state sequences

$$X_i = \Gamma_i^{\dagger} \mathcal{O}_i \tag{11}$$

6. Calculate A, B, B_0, C, D and D_0 solving the following set of equations:

$$\begin{bmatrix} A & [B\ B_0] \\ C & [D\ D_0] \end{bmatrix} \begin{bmatrix} X_i \\ U_{i|i} \end{bmatrix} = \begin{bmatrix} X_{i+1} \\ Y_{i|i} \end{bmatrix} \tag{12}$$

The state sequence X_{i+1} is calculated from the equation

$$X_{i+1} = \Gamma_{i-1}^{\dagger} \mathcal{O}_{i-1} \tag{13}$$

where Γ_{i-1} is the extended observability matrix Γ_i without the last l rows and $\mathcal{O}_{i-1}$ is the oblique projection

$$\mathcal{O}_{i-1} = Y_f^- /_{U_f^-} W_p^+ \tag{14}$$

The LRP of the fourth order:

$$A = \begin{bmatrix} 2.4032 & -2.0154 & 0.6586 & -0.0578 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0053 & 0.0112 & -0.0042 & -0.0009 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad B0 = \begin{bmatrix} -0.5714 \\ -0.1418 \\ -0.4527 \\ -0.2585 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix} \qquad D0 = \begin{bmatrix} 0.2 \end{bmatrix}$$

$y_0(p) = 0$, $p = 1, \ldots, 4 \times 10^5$; $x_{k+1}(0), k = 1, \ldots, 20$ – a uniformly distributed pseudorandom sequence on $[0, 2.5]$

Input $u_k(p)$ – a uniformly distributed pseudorandom sequence on $[0, 1]$

Output disturbance – a pseudorandom sequence of normal distribution with mean 0 and standard deviation 0.0001

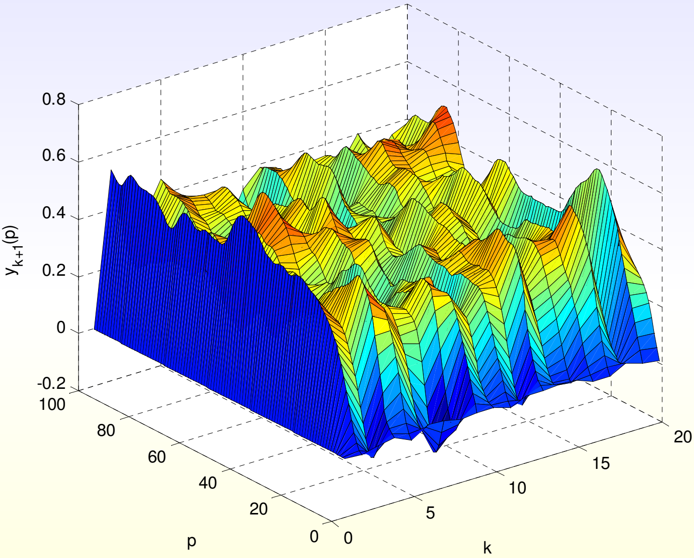

Fig. 1. Response of the LRP to the input $u_{k+1}(p)$.

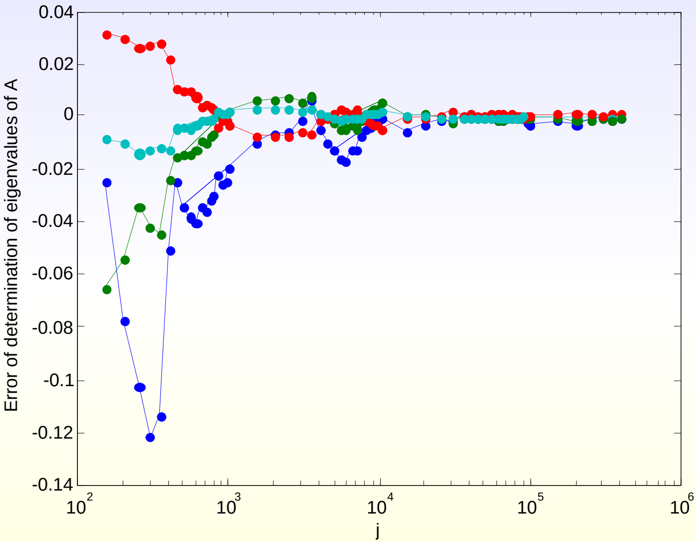

Fig. 2. Evolution of errors of determination of eigenvalues of A.

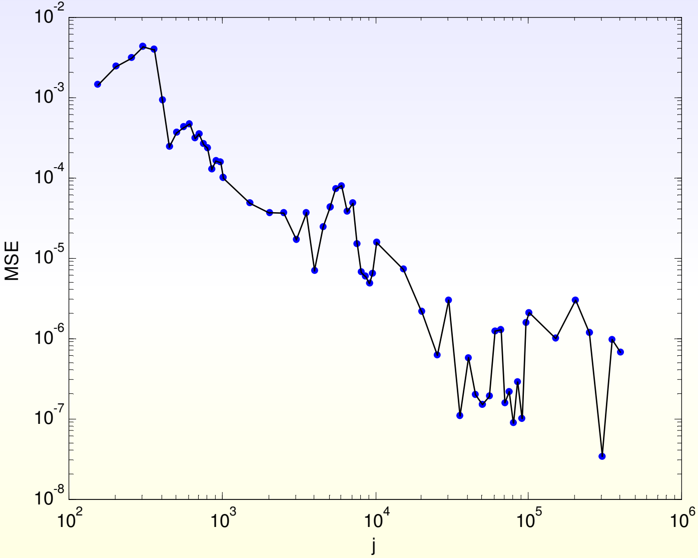

Fig. 3. MSE of determination of eigenvalues of A.

1. **A direct approach to the identification of LRPs based on subspace techniques is proposed**

2. **The unknown process matrices and the order of LRP are determined based on based on the input $u_{k+1}(p)$ and output $y_{k+1}(p)$ sequences of the actual pass and the output $y_k(p)$ sequence of the previous pass**

3. **The identification procedure can be restarted consecutively starting from the first pass $(k = 1)$ data and boundary conditions $y_0(p)$. Therefore, the proposed approach can be very useful not only for time invariant LRPs but also for processes with slowly evolving dynamics or processes which dynamics changes rapidly from pass to pass**

4. **Consistent estimates of eigenvalues in the case of additive output noise**