POLMAT library now within Symbolic Math Toolbox for Matlab in multidimensional systems computations

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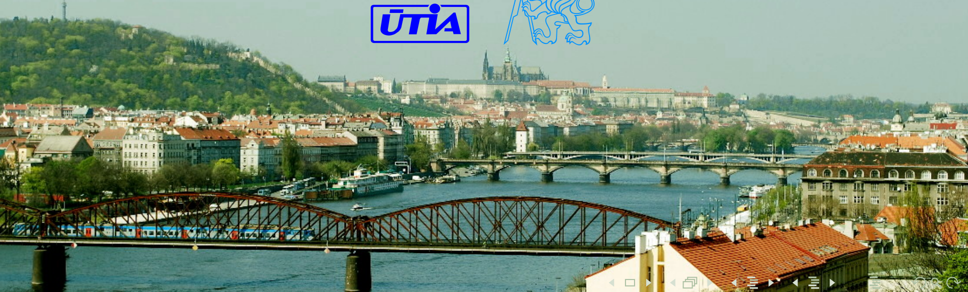


Outline

Aims of Polmat

- freely available library for symbolic computation with polynomial matrices
- developed for MuPAD, which was an easily accessible CAS
- focused on algorithms useful in control and filter design
- provides
  - algorithms for basic manipulations with polynomial matrices
  - solvers for linear equations with polynomial matrices
  - solvers for polynomial spectral factorisation
  - etc.

History of Polmat

2003 project was started

2004 Polmat website was found,
the first version of Polmat was introduced

2005 Polmat presented at IFAC World Congress

MuPAD ends providing academic licence

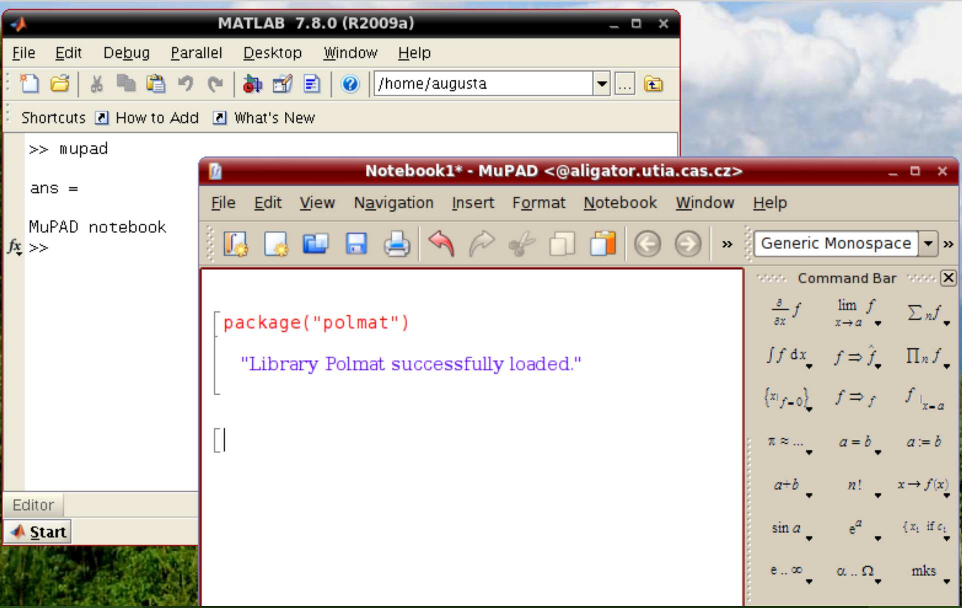
2008 MuPAD becomes part of
Symbolic math toolbox for Matlab 2008b

Polmat in multidimensional systems theory

We show

- manipulation with multivariate two-sided polynomials
- stability analysis of a spatially distributed system
- computation of the greatest common divisor of two-sided polynomials
- solving linear equations with multivariate polynomials

How to use Polmat



Basic properties of *n*-D polynomial

```
a:=s^2+(c/M)*s+(k/M)*(1-1/2/z-1/2*z)
```

$$s^2 + \frac{c\,s}{M} - \frac{k\,\left(\frac{z}{2} + \frac{1}{2\,z} - 1\right)}{M}$$

```
polmat::ldeg(a, s), polmat::tdeg(a, z),
polmat::ldeg(a, z)
```

$2,\ -1,\ 1$

```
polmat::tcoeff(a, s)
```

$$-\frac{k\,z^2 - 2\,k\,z + k}{2\,M\,z}$$

```
polmat::coeff(polmat::coeff(a, s, 0), z, 0)
```

$$\frac{k}{M}$$

Stability analysis

A long seismic cable

$$P(s,z) = \frac{b(s,z)}{a(s,z)} = \frac{1/M}{s^2 + (c/M)s + (k/M)[1 - 0.5\,z^{-1} - 0.5\,z]}$$

s – **temporal variable**, z – **spatial variable**

$k = 8$, $c = 10$ and $M = 40$

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```
a:=s^2+(c/M)*s+(k/M)*(1-1/2/z-1/2*z)
```

$$s^2 + \frac{c\,s}{M} - \frac{k\,(\frac{z}{2} + \frac{1}{2\,z} - 1)}{M}$$

```
a:=subs(a, [k=8, c=10, M=40])
```

$$\frac{s}{4} - \frac{z}{10} + s^2 - \frac{1}{10\,z} + \frac{1}{5}$$

Stability analysis
based on Hermite-Fujiwara matrix

$$a(s,z) = s^2 + \frac{1}{4}s - \frac{1}{10}(z + z^{-1}) + \frac{1}{5}$$

Stability condition

$$a(s,z) \neq 0, \quad \{\mathrm{Re}\, s \geq 0\} \cap \{|z| = 1\}$$

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$$H_a(1) \not\succ 0 \Rightarrow \text{system is not stable}$$

```
polmat::hermFuji(a, s)
```

$$\begin{pmatrix} -\frac{2z^2-4z+2}{80z} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$$

```
subs(%, z=1)
```

$$\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$$

```
linalg::isPosDef(%)
```

FALSE

Stability analysis

based on root map

```
p:=subs(solve(a, s), z=exp(I*o))
```

$$\left\{-\frac{\sqrt{\frac{\frac{1}{e^{o\,i}}\left(32+32\,e^{o\,2\,i}-59\,e^{o\,i}\right)}{80}}}{2}-\frac{1}{8},\ \frac{\sqrt{\frac{\frac{1}{e^{o\,i}}\left(32+32\,e^{o\,2\,i}-59\,e^{o\,i}\right)}{80}}}{2}-\frac{1}{8}\right\}$$

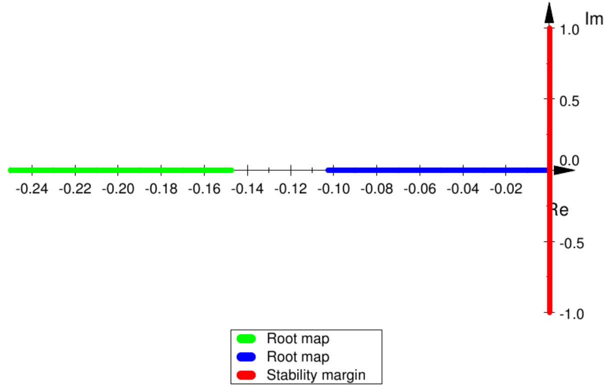
```
p1:=plot::Curve2d([Re(p[1]), Im(p[1])], o=0..2*PI,
Color = RGB::Green, LineWidth = 1,
Legend="Root map"):
```

```
p2:=plot::Curve2d([Re(p[2]), Im(p[2])], o=0..2*PI,
Color = RGB::Blue, LineWidth = 1,
Legend="Root map"):
```

```
p3:=plot::Curve2d([0, o], o=-1..1,
Color = RGB::Red, LineWidth = 1,
Legend = "Stability margin"):
```

Stability analysis
based on root map

```
plot(p1, p2, p3, AxesTitles = ["Re", "Im"])
```



The greatest common divisor

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$$a = (z^2 + z^{-2})\,(z + 2 + z^{-1}), \quad b = z + 2 + z^{-1}$$

```
a:=(z+2+1/z)*(z^2+1/z^2)
```

$$\left(\frac{1}{z^2} + z^2\right)\left(z + \frac{1}{z} + 2\right)$$

```
b:=(z+2+1/z)
```

$$z + \frac{1}{z} + 2$$

```
[g, p, q, r, s]:=polmat::gcd(a, b, z, all)
```

$$\left[z + \frac{1}{z} + 2,\ 0,\ 1,\ 1,\ -\frac{1}{z^2} - z^2\right]$$

Linear equations

Linear equation $ax + by = c$

given: $a(z_1, z_2, \ldots z_n)$, $b(z_1, z_2, \ldots z_n)$, $c(z_1, z_2, \ldots z_n)$
unknowns: $x(z_1, z_2, \ldots z_n)$, $y(z_1, z_2, \ldots z_n)$

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$a, b, c, x, y \in \mathcal{R}[z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n][z_i]$

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$$a, b, c, x, y \in \mathcal{R}[z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n][z_i]$$

$$a = 1 + z_1\,z_2 + z_3, \quad b = z_1\,z_2\,z_3, \quad c = z_1\,z_2 + z_1^2\,z_2^2$$

$$a, b, c \in \mathcal{R}[z_1, z_2][z_3]$$

Linear equations

```
a:=1+z[1]*z[2]+z[3]
```

$z_1\ z_2 + z_3 + 1$

```
b:=z[1]*z[2]*z[3]
```

$z_1\ z_2\ z_3$

```
c:=z[1]*z[2]+z[1]^2*z[2]^2
```

$z_1^2\ z_2^2 + z_1\ z_2$

```
[x,y]:=polmat::axbyc(a,b,c,z[3]):
```

```
[x,y]:=subs([x,y], t=0)
```

$[\mathrm{poly}(z_1\ z_2, [z_3]), \mathrm{poly}(-1, [z_3])]$

```
simplify(expr(a*(x+b*w)+b*(y-a*w)-c))
```

0

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`http://polmat.wz.cz`